Instructor: Dr. Rola Alseidi	Philadelphia University Faculty/College of Science Department of Basic Science amd Mathematics Midterm Exam	Academic Year: 2021/2022. Semester: First. Date: 29/11/2021. Course : General Topology. Duration of Exam: 75 minutes.
Name:		

- The exam consists of **3 pages**. Make sure you have all of them.

1. (5 points) Let $\mathbb{R}$ be the set of real numbers and let

$$\tau_{coc} = \{G \subseteq \mathbb{R} : G^c = \mathbb{R} \setminus G \text{ is countable}\} \cup \phi.$$

Show that τ_{coc} is a topology on $\mathbb{R}$.

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2. (5 points) Let $X = \{a, b, c\}$ and let $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$ be a topology on X. Find (**Show the details of your work**)

(a) The closed subsets of (X, τ)

(b) $\overline{\{a\}}$

(c) $\{b, c\}^0$

(d) $Bd(\{b, c\})$

(e) $Ext(\{b, c\})$

3. (12 points) Circle True or False. Read each statement carefully before answering and justify your answer

(a) True False Let τ and τ' be two topologies on the same set X. Then $\tau \cup \tau'$ is a topology on X.

(b) True False An arbitrary intersection of open sets in a topological space is open.

(c) True False In the topological space (X, τ_{dis}) the only dense set is X itself.

(d) True False Consider the topological space $(\mathbb{R}, \tau_u)$ and the set $A = [-1, 4]$. The set $(\frac{1}{2}, 5)$ is open in τ_α, the relative topology on the set A.

(e) True False Consider the topological space $(\mathbb{R}, \tau_u)$ and the topological space $(\mathbb{R}, \tau_\ell)$, τ_ℓ is finer than τ_u.

(f) True False Let τ and τ' be topologies on the same set X. If $\tau \subset \tau'$, then every closed set in τ is so in τ'.

(g) True False (X, τ) be a topological space and $A \subseteq X$, then the set $A^0 \cup Bd(A)$ is a closed set.

(h) True False The interval $[1, 3)$ is open in the topological space (X, τ_u).

4. (3 points) Let $X = \{a, b, c, d\}$ and $Y = \{1, 2, 3, 4\}$. We define

$$\tau_y = \{Y, \phi, \{3\}, \{1, 2, 3\}, \{3, 4\}\},$$

to be a topology on Y. Find the topology on X, (τ_x) induced by the topological space (Y, τ_y) and the following function

$$f = \{(a, 4), (c, 3), (d, 2)(b, 3)\}$$

Show the details of your work

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5. (5 points) Let (x, τ) be a topological space, $A, B \subseteq X$. Prove the following statements :
   (a) (2 points) If $A \subseteq B \Rightarrow A' \subseteq B'$.

   (b) (3 points) If $Bd(A) \cap A = \phi \Rightarrow A$ is an open set .

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6. (3 points) extra question
   Let

$$\tau = \{\{n, n+1, n+2, \dots\} : n \in \mathbb{N}\} \cup \phi.$$

   be a topology on $\mathbb{N}$.
   (a) Find the accumulation points of the set $A = \{4, 13, 28, 37\}$.

   (b) Find the closure of the set $A = \{7, 24, 47, 85\}$.